CONVENIENT TOPOLOGY

SOM NAIMPALLY

One of the most interesting results in Topology is that the category of Hausdorff k-spaces is convenient for all purposes of Topology. The concept of a convenient category was first introduced by Ronald Brown in two papers [3, 4].

However, in the literature it is often attributed to Norman Steenrod's 1967 paper [8]. This is rather strange since Brown's papers appeared in 1963, 1964 in the prestigious Oxford journal.

The introduction to the first article by Brown states, "It may be that the category of Hausdorff k-spaces is adequate and convenient for many major purposes of topology." In the second paper, Brown gives a list of properties satisfied by the category of Hausdorff spaces and k-continuous maps, and a footnote explains that these are also satisfied by the category of Hausdorff k-spaces, which is what Brown did in his Oxford D. Phil. thesis [2] (submitted 1961, approved 1962) which was mainly on the homotopy type of function spaces, and which Brown circulated to many universities including Princeton. It was *well-known* at the time of Steenrod's paper how to do the non-Hausdorff case (take final topologies). This is explained in the 1988 edition of Brown's book [5].

There is another mathematical point that is worth making, namely that in Brown's paper, using k-continuous functions, he gets a homeomorphism for the exponential law, whereas Steenrod gets only a homeomorphism on the k-ifications. So Brown's results are stronger. Also Brown's paper has more results in this context of exponential laws, though this may have distracted from the *convenient* aspects.

The term *convenient category* caught on, as is now known—see for example the book by A. Kriegl and P. Michor [6]. This book is good on the history—and Brown was quite aware in his D. Phil. thesis of what was the purpose of this change of paradigm, since looking at examples of what were later called Cartesian closed or monoidal closed categories was what made the Thesis work.

The subject has now blossomed into a grand theory with the infusion of convergence and uniform convergence structures, semiuniform convergence spaces etc. An up-to-date account is written in the recently published book by Gerhard Preuss [7].

References

[1] P. Booth and J. Tillotson, *Monoidal closed, Cartesian closed and convenient categories of topological spaces*, Pacific J. Math. **88** (1980), no. 1, 35–53. MR 82a:55021 Zbl 0402.54004

[2] Ronald Brown, *Some problems in algebraic topology: function spaces and FD complexes*, D. Phil. thesis, University of Oxford, 1962.

[3] ———, *Ten topologies for* $X \times Y$, Quart. J. Math. Oxford Ser. (2) **14** (1963), 303–319. MR 28 #2516 Zbl 0113.37504

Published in Topological Commentary **8** no. 1 (2003).
http://at.yorku.ca/topology/topcom.htm .

[4] ______, *Function spaces and product topologies*, Quart. J. Math. Oxford Ser. (2) **15** (1964), 238–250. MR 29 #2779 Zbl 0126.38503

[5] ______, *Topology: a geometric account of general topology, homotopy types and the fundamental groupoid*, second ed., Ellis Horwood Ltd., Chichester, 1988. MR 90k:54001 Zbl 0655.55001

[6] Andreas Kriegl and Peter W. Michor, *The convenient setting of global analysis*, Mathematical Surveys and Monographs, vol. 53, American Mathematical Society, Providence, RI, 1997. MR 98i:58015 Zbl 0889.58001

[7] Gerhard Preuss, *Foundations of topology: an approach to convenient topology*, Kluwer Academic Press, Dordrecht, 2002.

[8] Norman E. Steenrod, *A convenient category of topological spaces*, Michigan Math. J. **14** (1967), 133–152. MR 35 #970 Zbl 0145.43002

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