

**SUPERNEARNESS SPACES AS A TOOL FOR STUDYING
UNIFICATION AND EXTENSIONS**

D. LESEBERG

Topological extensions are closely related to nearness structures of various kinds. For example, the Smirnov compactification of a proximity space X is a compact Hausdorff space Y which contains X as a dense subspace and for which it is true that a pair of subsets of X is near if, and only if, their closures in Y meet. Lodato generalized this result to weaker conditions for the proximity and the space using *bunches* for the characterization of the extension.

Ivanova and Ivanov studied contiguity spaces and bicomact extensions such that a finite family of subsets are contiguous if, and only if, there is a point of Y which is simultaneously in the closure in Y of each set of the family. Herrlich found a useful generalization of contiguity spaces by introducing nearness spaces, and Bentley showed that those nearness spaces which can be extended to topological ones have a neat internal characterization. Doitchinov introduced the notion of super-topological spaces in order to construct a unified theory of topological, proximity and uniform spaces, and he proved a certain relationship of some special classes of supertopologies—called b -supertopologies—with compactly determined extensions. Recently, supernear spaces were introduced by myself in order to define a common generalization of nearness spaces and supertopological spaces as well. As a basic concept, the notion of bounded sets is used, and functions (supernear operators) from a collection of bounded sets into powers of a given set are considered which fulfill certain axiom of nearness. So, we define for each bounded set the so-called *B-near collections*, and a bounded map is defined as a supernear-map if it preserves such collections. Thus, the new topological category **SN** whose objects are the supernear spaces is established, and moreover we can characterize those supernear spaces which can be extended to topological ones. As a basic notion we consider topological extensions which are strict in the sense of Banaschewski, where the hull in Y of the images of sets in X form a base for the closed subsets in Y . In this connection we study symmetrical and non-symmetrical extensions, and consequently we can characterize and describe those supernear spaces which are in a certain one-to-one relationship or correspondence, respectively, with them.

In other words, we describe those supernear spaces, whose B -near collections are determined by their respective constructing strict topological extension. As special cases we get back the former described constructions by various authors. So it seems to be possible to describe topological unification and extensions by the support of one *topological concept* with its corresponding maps.

Date: October 2003.

2000 Mathematics Subject Classification. 54A05, 54A20, 54C35, 54D35 54E15.

Key words and phrases. topological constructs; posets; strict extensions; supernearness; supertopologies; nearness; compactly determined; grill-determined; clan-determined.

REFERENCES

- [1] B. Banaschewski, *Extensions of topological spaces*, Canad. Math. Bull. **7** (1964), 1–22. MR 28 #4501 Zbl 0121.17304
- [2] H. L. Bentley, *Nearness spaces and extensions of topological spaces*, Studies in topology (Charlotte, N.C., 1974), Academic Press, New York, 1975, pp. 47–66. MR 52 #1635 Zbl 0314.54013
- [3] Á. Császár, *Foundations of general topology*, Pergamon Press, Oxford-London-New York-Paris, 1963. MR 28 #575 Zbl 0108.35304
- [4] D. Doitchinov, *A unified theory of topological spaces, proximity spaces and uniform spaces*, Dokl. Akad. Nauk SSSR **156** (1964), 21–24, English translation: Sov. Math. Dokl. **4** (1964) 595–598. MR 29 #5221 Zbl 0137.42101
- [5] ———, *Compactly determined extensions of topological spaces*, Serdica **11** (1985), no. 3, 269–286. MR 87d:54025 Zbl 0591.54016
- [6] W. Gähler, *Extension structures and completions in topology and algebra*, Seminarberichte aus dem Fachbereich Mathematik der Fernuniversität Hagen **70** (2001), 77–103.
- [7] E. Hayashi, *On some properties of a proximity*, J. Math. Soc. Japan **16** (1964), 375–378. MR 31 #2708 Zbl 0137.42102
- [8] H. Herrlich, *A concept of nearness*, General Topology and Appl. **4** (1974), 191–212. MR 50 #3193 Zbl 0288.54004
- [9] M. Hušek, *Categorical connections between generalized proximity spaces and compactifications*, Contributions to Extension Theory of Topological Structures (Proc. Sympos., Berlin, 1967), Deutsch. Verlag Wissensch., Berlin, 1969, pp. 127–132. MR 40 #1981 Zbl 0182.25204
- [10] V. M. Ivanova and A. A. Ivanov, *Contiguity spaces and bicomact extensions of topological spaces*, Dokl. Akad. Nauk SSSR **127** (1959), 20–22 (Russian). MR 21 #5943 Zbl 0091.35902
- [11] D. Leseberg, *A new concept of nearness*, Contributions to general algebra, 13 (Velké Karlovice, 1999/Dresden, 2000), Heyn, Klagenfurt, 2001, pp. 207–217. MR 2002f:54019 Zbl 0994.54001
- [12] ———, *Ordered continuity structures and their relationship to topology*, Demonstratio Math. **35** (2002), no. 1, 175–197. MR 2003b:54011 Zbl 1007.54001
- [13] ———, *Supernearness, a common concept of supertopologies and nearness*, Topology Appl. **123** (2002), no. 1, 145–156. MR 2003f:54015 Zbl 1002.54019
- [14] ———, *Symmetrical extensions and generalized nearness*, 2002, Proceedings of the workshop on Coverings, Selections and Games in Topology (Lecce), to appear.
- [15] ———, *Compact extensions and contigual supernearness*, The Pumplün 70 Festschrift, RWTH Aachen, 2003. <http://www.matha.rwth-aachen.de/publikationen/pumpluen-festschrift>
- [16] ———, *Supertopologies as starting points for generalized continuity structures*, 2003, Proceedings of NWECS 2003 (Berlin), to appear.
- [17] M. W. Lodato, *On topologically induced generalized proximity relations*, Proc. Amer. Math. Soc. **15** (1964), 417–422. MR 28 #4513 Zbl 0129.37604
- [18] ———, *On topologically induced generalized proximity relations. II*, Pacific J. Math. **17** (1966), 131–135. MR 33 #695 Zbl 0138.17704
- [19] S. A. Naimpally and B. D. Warrack, *Proximity spaces*, Cambridge University Press, London, 1970. MR 43 #3992 Zbl 0206.24601
- [20] Yu. Smirnov, *On the completeness of proximity spaces*, Doklady Akad. Nauk SSSR (N.S.) **88** (1953), 761–764 (Russian). MR 15,144c Zbl 0050.17002
- [21] A. D. Taïmanov, *On extension of continuous mappings of topological spaces*, Mat. Sbornik N.S. **31(73)** (1952), 459–463 (Russian). MR 14,395a Zbl 0048.16804
- [22] W. L. Terwilliger, *On contiguity spaces*, Ph.D. thesis, Washington State University, 1965.
- [23] A. Tozzi and O. Wyler, *On categories of supertopological spaces*, Acta Univ. Carolin. Math. Phys. **28** (1987), no. 2, 137–149. MR 89b:54014 Zbl 0651.54004

DIETER LESEBERG, DEPARTMENT OF MATHEMATICS AND INFORMATICS, FREE UNIVERSITY OF BERLIN, GERMANY

E-mail address: d.leseberg@tu-bs.de