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A GENERAL CLASS OF INTEGRAL OPERATORS PRESERVING SUBORDINATIONS AND SUPERORDINATIONS

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If $H(\mathrm{U})$ denotes the space of analytic functions in the unit disk U, for the integral operator $A^h_{\alpha,\beta,\gamma} : \mathcal{K} \to H(\mathrm{U})$, with $\mathcal{K} \subset H(\mathrm{U})$, defined by

$$A^h_{\alpha,\beta,\gamma}[f](z) = \left[\frac{\beta+\gamma}{z^\gamma}\int_0^z f^\alpha(t)h(t)t^{\delta-1}\,\mathrm{d}\,t\right]^{1/\beta},$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $h \in H(\mathrm{U})$, we determined sufficient conditions on g_1, g_2, α, β and γ such that

$$zh(z)\,[g_1(z)/z]^\alpha \ll zh(z)\,[f(z)/z]^\alpha \ll zh(z)\,[g_2(z)/z]^\alpha$$

implies

$$z\left[A^h_{\alpha,\beta,\gamma}[g_1](z)/z\right]^\beta \ll z\left[A^h_{\alpha,\beta,\gamma}[f](z)/z\right]^\beta \ll z\left[A^h_{\alpha,\beta,\gamma}[g_2](z)/z\right]^\beta,$$

where the symbol " $\ll$ " represents the *subordination*.

In addition, both of the subordinations are sharp, since $z\left[A^h_{\alpha,\beta,\gamma}[g_1](z)/z\right]^\beta$ is the *largest* function and $z\left[A^h_{\alpha,\beta,\gamma}[g_2](z)/z\right]^\beta$ is the *smallest* function so that the left-hand side, respectively the right-hand side of the above implication hold, for all f functions satisfying the differential subordination, respectively the differential superordination of the assumption. The results generalizes those of the last author, obtained for the special case $\alpha = \beta$ and $h \equiv 1$.

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