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THE FATOU INEQUALITY AND WANDERING CONTINUA

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Let P be a polynomial of degree d with Julia set J_P. Let N be the number of cycles of bounded Fatou domains of P plus the number of Cremer periodic orbits of P. Then the number of attracting and neutral periodic cycles of P is less than or equal to N. The famous Fatou-Douady-Hubbard-Shishikura inequality states that $N \leq d-1$. The main goal of this paper is to improve this bound.

Denote by N_∞ the number of repelling periodic orbits at which infinitely many external rays land (such orbits do not exist if J_P is connected). A *wandering collection* is a collection of continua/points $W \subset J_P$ with pairwise disjoint images. It is known that wandering collections exist.

We show that if $W \subset E$, where E is a component of J_P, and $E \setminus W$ is empty or disconnected, then there is a finite set $A(W) \neq$ of external rays with principal sets in W. Call W *non-precritical* if $\sigma_d^n|_{A(W)}$ is one-to-one for any n ($\sigma_d = z^d|_{S^1} : S^1 \to S^1$ is the map acting on the angles in the circle at infinity). Given a *non-empty* wandering collection Γ of *non-precritical* continua/points $Q_1, \ldots, Q_k$ in J_P with $|A(Q_i)| > 2$ $(1 \leq i \leq k)$, we prove that

$$\sum_{\Gamma}(|A(Q_i)|-2) + N + N_\infty \leq d-2.$$

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