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WHEN IS THE ISBELL TOPOLOGY A GROUP TOPOLOGY?

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The Isbell topology on the set $C(X)$ of real-valued continuous functions on a topological space X has a subbase formed by sets of the form

$$[\mathcal{A}, U] := \{f \in C(X) : \exists A \in \mathcal{A}, f(A) \subseteq U\},$$

where $\mathcal{A}$ ranges over the Scott open subsets of the space $\mathcal{O}(X)$ of open subsets of X and U ranges over open subsets of $\mathbb{R}$. Scott open sets $\mathcal{A}$ are also called *compact families*. They are closed under open supersets, and satisfy

$$\mathcal{S} \subseteq \mathcal{O}(X), \cup_{S\in\mathcal{S}} S \in \mathcal{A} \Longrightarrow \exists \mathcal{F} \subseteq \mathcal{S}, |\mathcal{F}| < \infty, \cup_{S\in\mathcal{F}} S \in \mathcal{A}.$$

Conditions under which the Isbell topology is a group topology are studied. Of course, when X is consonant, every compact family is generated by compact subsets and the Isbell topology coincides with the compact-open topology, and is therefore a group topology. We obtain charaterizations for the continuity of translations on one hand, and for the joint continuity of addition at the zero function on the other, yielding conditions for the Isbell topology to be a group topology that are formally weaker than consonance. However, an example of a non consonant space X for which the Isbell topology of $C(X)$ is a group topology remains elusive so far.

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