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LOGICS OF QUANTUM COMPUTATION WITH QUANTUM SIMULABLE OPERATORS

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Abstract In quantum computational logics meaning of sentences are represented by quantum information quantities.
Any *n-qubit* sequence is identified with a unit vector of the tensor product of the complex Hilbert space $\otimes^n \mathrm{C}^2$.
Let us split the set of the vectors of its computational basis in the two sets: $B_1^n := \{i :\| i \gg =| x_1, .., x_n\rangle \text{ and } x_n = 1\}$; $B_0^n := \{i :\| i \gg =| x_1, .., x_n\rangle \text{ and } x_n = 0\}$. Any unit vector $| \psi\rangle$ can be expressed in the form:
$| \psi\rangle = \sum_{i\in B_0} a_i \| i \gg + \sum_{j\in B_1} b_j \| j \gg$ where by the Born rule,
$\sum_{i\in B_0} | a_i |^2 + \sum_{j\in B_1} | b_j |^2 = 1$
A key role in the definition of semantics is played by the probability-value of $| \psi\rangle$, defined as follows:
$\mathrm{p}(| \psi\rangle) := \sum_{j\in B_1} | b_j |^2$
Moreover a function $g : [0, 1]^2 \mapsto [0, 1]$ is said quantum simulable iff $\exists n \geq 1$, a unitary operator $U_g : \otimes^{n+2} C^2 \mapsto \otimes^{n+2} C^2$ and a quregister $| \chi\rangle \in \otimes^n C^2$ such that for any pair $| \psi\rangle$, $| \phi\rangle$ of qubits in C^2, the following condition is satisfied:
$\mathrm{p}(U_g(| \psi\rangle | \phi\rangle | \chi\rangle)) = g(\mathrm{p}(| \psi\rangle), \mathrm{p}(| \phi\rangle))$.
A theorem by Dalla Chiara, Giuntini and Leporini shows there are exactly 16 quantum simulable fuzzy extensions of the 16 non-equivalent binary boolean functions. Thus I study the algebra of the unit interval of reals endowed with product (i.e. the unique quantum simulable fuzzy extension of AND) and involution (all the other extensions can be further defined). I prove the variety generated by this algebra can be axiomatized by only six equations. Then I define a structure in which an intrinsic unitary quantum operator (i.e. square root of negation) is added. Some results about the properties of this new class of algebras are presented and the related arising logics is studied.

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