An $\alpha\gamma$ algebra is a residuated lattice satisfying conditions:

(C): \((xy)(yz) = yx\) and \((C_\wedge)\): \(x \wedge y = [x \odot (xy)] \lor [y \odot (yx)]\),

while an $\alpha$ algebra ($\gamma$ algebra) is a residuated lattice satisfying condition (C) \((C_\wedge)\) respectively. Recall that a BL algebra is a bounded residuated lattice satisfying conditions:

(pre): \((xy) \lor (yx) = 1\) and (div): \(x \land y = x \odot (xy)\),

while a MTL algebra (bounded divisible residuated lattice = bounded commutative RL-monoid) is a bounded residuated lattice satisfying condition (pre) ((div) respectively).

We get: (pre) $\iff$ \((C) + (C_\lor) \iff (C_\wedge) + (C_{\epsilon})\) and (div) $\iff$ \((C) + (C_\delta) \iff (C_\wedge) + (C_{\pi})\),

where \((C_\lor)\): \(x \lor y = [(xy)y] \land [(yx)x]\) and the independent conditions \((C_{\epsilon}), (C_\delta), (C_{\pi})\) must be found (open problem). It follows that: (1) bounded $\alpha\gamma$ algebras are a common generalization of MTL algebras and of bounded divisible residuated lattices; (2) the MTL algebras with condition (DN) (Double Negation): for all \(x\), \((x^-)^- = x\), and the bounded $\alpha\gamma$ algebras with condition (DN) are the IMTL algebras (just like the BL algebras with condition (DN) and the divisible bounded residuated lattices with condition (DN) are the MV algebras); therefore, we have obtained classes of examples of MTL algebras and of bounded $\alpha\gamma$ algebras by starting with IMTL algebras and by using the ordinal product; (3) the ordinal product of two proper bounded $\alpha\gamma$ algebras is again a proper bounded $\alpha\gamma$ algebra; (4) the ordinal product: linearly ordered MTL (BL) algebra $\bowtie$ MTL (BL) algebra is again a MTL (BL) algebra, while the ordinal product: not-linearly ordered MTL (BL) algebra $\bowtie$ MTL (BL) algebra is only a bounded $\alpha\gamma$ algebra (bounded divisible residuated lattice).

We give classes of examples of finite proper IMTL algebras, MTL algebras and bounded $\alpha\gamma$ algebras, satisfying or not satisfying condition (WNM) (Weak Nilpotent Minimum): for all \(x, y\), \((x \odot y)^- \lor [(x \land y)(x \odot y)] = 1\).

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