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RUNGE-KUTTA DISCONTINUOUS GALERKIN SCHEMES FOR A MODEL CONSTRAINED EVOLUTION PROBLEM

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Problems of constrained evolution, that is problems comprising equations both in time and space derivatives (called evolution equations) and equations that have spatial derivatives only (called constraint equations), are encountered in electromagnetism, computational fluid dynamics, and very recently, the numerical general relativity. In both electromagnetism and relativity, the evolution equations have the property of preserving the constraint equations. Namely, if boundary is absent, any solution to the evolution equations that satisfies the constraint equations initially will automatically satisfy the constraint equations for all times. This property of the equations has largely motivated the approach of free evolution used both in electromagnetism and relativity when one only solves the evolution equations while the constraint equations are monitored but not actively enforced. One should not expect, however, that the constraint equations are continuing to be preserved in the discretized systems. In fact, in simulations one has to worry about the growth of the initially small constraint violations. The problem becomes even more complex when an artificial boundary is introduced, since now the constraint violating modes can be injected into the domain through the boundary. The problem of constraint-compatible boundary conditions must then be introduced and studied to guarantee that the free evolution problem yields a physical solution. The expected boundary conditions must be compatible with the constraint equations (they must not perturb the constraint quantity), they must yield a well-posed initial-boundary value problem (for the free evolution), and, additionally, they must minimize spurious reflections of radiation from the boundary. The latter property is important for the domain decomposition and the conditions on the artificial outer boundary.

In this work we investigate the numerical properties of the radiation-controlling constraint-compatible boundary conditions obtained earlier by the author for the model problem of vector wave equation subject to the divergence-free constraint. We introduce a Runge-Kutta in time and discontinous Galerkin in space discretization of the second-order wave equation. We compare different weak formulations of the problem and discuss the effects of the formulation on the stability and convergence properties of the system as well as on its constraint preservation property. Two first order symmetric hyperbolic reductions, one based on the div-grad and another based on the curl-curl + grad-div decomposition of the wave operator are studied and the results are compared to the second-order wave equation. Stability of the problem with respect to perturbations in boundary conditions is studied.

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