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APPLICATIONS OF BANACH-VALUED BESOV SPACES TO EVOLUTION EQUATIONS IN BANACH SPACES

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It is well known that Besov spaces are very usefull in studying many problems. I report here their applications to the evolution equation: (EE) $du/dt + A(t)u = f(t), a < t < b$, where $-A(t)$ is the generator of a semigroup of linear operators in a Banach space X. We consider only the case where $-A(t)$ is the generator of an analytic semogroup.

Case 1. $A(t) = A$ is independent of t. Crandall-Pazy, 1969, proved that $F(t) := \int_a^t e^{-(t-s)A} f(s)ds(e^{-tA}$ denotes the semigroup generated by $-A)$ is strongly differentiable and satisfies (EE) if the modulus of continuity $\omega(h : f)$ of f is integrable near 0 with the measure dh/h. Furthermoere, Baillon, 1980, showed that if F is differantiable for every continuous function f then X has a spacial property or A is bounded. We prove that F is strongly differantiable and satisfies (EE) if f belongs to $B^0_{\infty,1}(I; X)_{loc} \cap L^1(I; X), I := (a, b)$ (J.Math.Soc. Japan, 1990).

Case 2. The domain $D(A(t))$ of $A(t)$ is independent of t, which we write by Y. Tanabe, 1960, has constructed the evolutin operator $U(t, s)$ to (EE) when $A(t)$ is Hölder continuous $L(Y, X)$-valued function. We have improved his result, that is, we have constructed it under the assumption that the modulas of continuity $\omega(h)$ of $A(t)$ as an $L(Y, X)$-valued function is integrable near 0 with dh/h(Osaka J. Math. 2001).We also showed that $F(t) := \int_a^t U(t, s)f(s)ds$ is strongly differentiable and satisfies (EE) if f satisfies the same conditon as in Case I.

Case 3. The domain $D(A(t)^{1/m})$ is independent of t, where m is some positive integer m greater than 1. We put $Y = D(A(t)^{1/m})$. Assuming that $A(t)^{1/m}$ is Hölder continuous with a exponent θ greater than $1 - 1/m$ as an $L(Y, X)$-valued function, T. Kato, 1961, has constructed the evolution operator. We recently improved his result. Our assumption is '$A(t)$ belongs to $B^{1-1/m}_{\infty,1}(I; L(Y, X))$'. We also have the same result for F as in Case 2.

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