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Coauthors: A.A. Kilbas

GENERALIZED EXTENDED MITTAG-LEFFLER TRANSFORM

A.A. KOROLEVA

The generalized extended Mittag-Leffler function $\mathcal{E}_{\alpha_1,\beta_1,\alpha_2,\beta_2}(z)$ with complex $\alpha_1,\,\alpha_2 \in \mathbb{C}\,(\alpha_1+\alpha_2 \neq 0)$ and $\beta_1, \beta_2 \in \mathbb{C}$ is defined by the following Mellin-Barnes integral

$$\mathcal{E}_{\alpha_1,\beta_1,\,\alpha_2,\beta_2}(z) = \frac{1}{2\pi i}\int\limits_{\mathcal{L}} \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(\beta_1-\alpha_1 s)\Gamma(\beta_2-\alpha_2 s)}(-z)^{-s}ds \quad (z \neq 0). \tag{1}$$

Our report deals with the integral transform

$$(\mathcal{E}_{\alpha_1,\beta_1,\,\alpha_2,\beta_2}f)(x) = \int\limits_0^\infty \mathcal{E}_{\alpha_1,\beta_1,\,\alpha_2,\beta_2}(-xt)f(t)dt \quad (x>0), \tag{2}$$

containing the function (1) in the kernel. We show that this transform is a special case of the so-called H-transform [2]. On the basis of this fact we establish mapping properties such as the boundedness, the representation and the range of the transform (2) and its inversion formulas in the space $\mathcal{L}_{\nu,\,r}$ of the Lebesgue measurable functions f on the $\mathbb{R}_+ = (0,\infty)$ such that

$$\int\limits_0^\infty |t^\nu f(t)|\,\frac{dt}{t} < \infty \quad (1 \le r \le \infty, v \in \mathbb{R}). \tag{3}$$

Similar properties for the transform of the form (2), in which $\mathcal{E}_{\alpha_1,\beta_1,\alpha_2,\beta_2}(z)$ is replaced by the classical Mittag–Leffler functions [3, Sect. 18.1]

$$E_{\alpha,\,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)} \;\; (\alpha>0; \beta\in\mathbb{R}; z\in\mathbb{C}) \tag{4}$$

with $\alpha > 0$, were given in [1].

References

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Assistent Professor, Belarusian State University, Minsk 220050, Belarus
E-mail address: akoroleva@tut.by