Automorphisms of powers of linearly ordered sets

Elbert Walker
elbert@nmsu.edu
with Carol Walker
hardy@nmsu.edu

Department of Mathematical Sciences, New Mexico State University
Las Cruces, NM 88003, USA

Let $\mathbb{I} = ([0,1], \vee, \wedge, 0, 1)$, where $\vee$ and $\wedge$ are max and min, respectively. This algebra is the basic building block of fuzzy set theory and logic. Likewise, the algebra $\mathbb{I}^{[2]} = ([0,1]^{[2]}, \vee, \wedge, 0, 1)$, where $[0,1]^{[2]} = \{(a,b) : a, b \in [0,1],\ a \leq b\}$, $\vee$, $\wedge$ are given coordinate-wise, and 0 and 1 are the bounds on $[0,1]^{[2]}$, is the relevant algebra for interval-valued fuzzy set theory and logic. A good share of the theory of fuzzy sets is concerned with endowing $\mathbb{I}$ with additional structure such as t-norms, t-conorms, and negations other than the usual max and min operations, and usual negation.

In addressing the situation for $\mathbb{I}^{[2]}$, a basic problem was deciding on the appropriate definitions [1]. For example, what should a t-norm on $\mathbb{I}^{[2]}$ be? In this regard, knowing the automorphisms of $\mathbb{I}^{[2]}$ was fundamental for representation theorems for norms, conorms, and negations. The basic theorem is that any automorphism of $\mathbb{I}^{[2]}$ is of the form $(a,b) \to (\varphi(a), \varphi(b))$ where φ is an automorphism of $\mathbb{I}$ [1].

Our initial motivation was to extend this theorem from $\mathbb{I}^{[2]}$ to $\mathbb{I}^{[n]}$ for any positive integer n. But more generally, we will replace $[0,1]$ by a bounded linearly ordered set S, and consider the automorphisms of both $S^{[n]}$ and S^n. The result for $S^{[n]}$ is the same as for $\mathbb{I}^{[2]}$ except for an anomaly for certain combinations of finite S and integers n.

References

[1] Some Comments on Interval Valued Fuzzy Sets, *International Journal of Intelligent Systems* 11(1996) 751-759 (with M. Gehrke).