Principal fuzzy type theories as higher order fuzzy logics

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The successful development of the formal theory of fuzzy logic started in 1979 by the seminal paper of J. Pavelka [10]. He developed the propositional fuzzy logic. Its first order version has been developed by V. Novák in [6] (see also [9]). This logic is based on Łukasiewicz MV-algebra of truth values and has evaluated syntax. Since the book of P. Hájek [4] appeared, a rapid development of various kinds of formal fuzzy logics differing in the used structures of truth values can be noticed (see also [8]).

Recently, fuzzy logic penetrated also to higher order, and a formal theory of *fuzzy type theory* has been developed, first in [7] and in a slightly different form also in [2].

The paper [7] follows the development of the classical type theory, as elaborated by A. Church [3] and L. Henkin [5], and later continued, e.g. in [1].

Because of a large variety of possibilities, a discussion about what is fuzzy logic is now in progress. Based on the expressive power and various experiences, several distinguished kinds of fuzzy logic took privilege over the other ones. Such logics are Łukasiewicz, basic fuzzy logic (BL) and LΠ fuzzy logic. There are several reasons for this fact. However, it turns out that also fuzzy type theory can be developed in parallel ways. The original theory in [7] has been developed on the basis of -algebra that is, a residuated lattice with prelinearity and double negation extended, moreover, by a special unary operation of Baaz delta. In this paper we will present all these kinds of fuzzy type theory, namely those based on IMTL_Δ, Łukasiewicz$_\Delta$, BL_Δ and LΠ algebras. These theories enjoy the generalized completeness property (i.e. completeness w.r.t. generalized models). It should be stressed that the fundamental connective in all of them is a fuzzy equality. Because of essential importance of this connective, the resulting theory is elegant and philosophically interesting.

We will present logical axioms, inference rules, semantics, and some specific properties of all four kinds of fuzzy type theory including the completeness theorems.

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