

Weakly algebraizable Gentzen systems

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A deductive system (Hilbert-style) is an algebraic invariant closure system over the set of formulas of a given propositional language, where invariant means that it is closed under inverse substitutions. Similarly, a Gentzen system can be seen as an algebraic invariant closure system over the set of all sequents, i.e., finite sequences of formulas, of this language. (Substitutions act on sequents componentwise.) The main result of the work is a technique that allows us to adapt the methods, previously developed in the area of algebraic logic for Hilbert deductive systems, to the case of Gentzen systems. Using the properties of the Tarski congruence, a generalization of the Leibniz congruence, we develop an algebraic hierarchy for Gentzen systems that closely parallels the well-known algebraic hierarchy of Hilbert deductive systems. This approach allows us to unify in a single framework several previously known results about algebraizable and equivalential Gentzen systems. We also obtain a characterization of weakly algebraizable Gentzen systems.

The significance of Gentzen systems and related axiomatizations by Gentzen rules is due in large part to the fact that various metatheoretical properties of Hilbert deductive systems can be formulated in terms of Gentzen systems. In particular, it was observed that a number of important non-protoalgebraic deductive systems that have a natural algebraic semantics also have so-called fully adequate Gentzen systems associated with them, the conjunction-disjunction fragment of the classical propositional logic being a paradigmatic example. Using the fact that any fully adequate Gentzen system is weakly algebraizable in our sense, we formulate a general criterion for the existence of a fully adequate Gentzen system, which works both for protoalgebraic and non-protoalgebraic Hilbert deductive systems, and show that many of the known partial results can be explained based on this general criterion. This includes such cases as the existence of fully adequate Gentzen systems for self-extensional logics with conjunction or implication, and the criteria for the existence of a fully adequate Gentzen system for protoalgebraic and weakly algebraizable logics.

References

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